Clock Adjustments for Neutrino Velocity Measurement

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# Contents

Abstract........................................................................................................................................3

1 Introduction ..................................................................................................................................4

2 Einstein’s Train Thought-Experiment .........................................................................................4

2.1 The Relativity of Simultaneity ................................................................................................4

2.2 Adjusting Clocks Between Different Frames of Reference......................................................6

2.3 Multiple Frames of Reference ...............................................................................................7

3 The CMB as a Stationary Frame of Reference .........................................................................7

4 Implications for the OPERA Neutrino Speed Measurement ...................................................9

4.1 GPS Measurements .................................................................................................................9

4.2 Two-Way Light Speed Measurements at CERN and OPERA .............................................9

5 Correcting the OPERA Neutrino Speed Measurements ............................................................11

5.1 Common Component of Excess Neutrino Velocity ...............................................................11

5.2 Variable Component of Excess Neutrino Velocity ................................................................11

5.3 Possible Errors in Other Experiments ....................................................................................12

References......................................................................................................................................13

Modification History......................................................................................................................14
Abstract
The OPERA neutrino experiment at the Gran Sasso Laboratory, [1], obtained a measurement, \( v \), of the neutrino muon velocity with respect to the speed of light, \( c \), of \( (v-c)/c = (2.48 \pm 0.28 \text{ (stat.)} \pm 0.30 \text{ (sys.)}) \times 10^{-5} \), that is, in excess of \( c \) by about 1 part in 40,000. Over most of the neutrino flight path from CERN to OPERA, distances and timings were established by GPS signals to within 2cm and \( (2.3 \pm 0.9)\text{ns} \) respectively. These measurements are not problematic. The Gran Sasso Laboratory is underground, however, and a significant part of the measurement of the expected flight time at \( c \) was established using two separate fibre delay calibration procedures, both of which are based on the two-way speed of light rather than the reference-frame dependent one-way speed implicit in the GPS and the velocity of neutrinos over the (one-way) flight path from CERN to OPERA. This ignores the moving frame of reference (the Earth system (ES)) in which the experiment was conducted and introduces errors of the same order as the early arrival time of the neutrinos \( (60.7 \pm 6.9 \text{ (stat.)} \pm 7.4 \text{ (sys.)}) \text{ ns} \). Similar problems affect all attempts to measure the one-way speed of light [2]. This paper explains these problems, with reference to Einstein’s train thought-experiment and suggests how the expected one-way speed of light in a moving frame of reference such as the ES could be derived using the cosmic microwave background (CMB) as a stationary frame of reference. This problem is additional to the clock synchronisation issues described in [3], and can be traced to a flaw in a procedure, [5], which may have been used at CERN on other occasions, so may affect other experiments involving the accurate measurement of particle velocities approaching or at \( c \).
1 Introduction

The OPERA neutrino experiment at the underground Gran Sasso Laboratory measured the velocity of neutrinos from the CERN CNGS beam over a baseline of about 730 km and compared this to the expected speed of light, c, in a vacuum [1]. Accurate measurements of c have only ever been successfully measured in two-way experiments. The reason for this is clear from Einstein’s train thought-experiment. A careful consideration of the train thought-experiment suggests a one-way light speed can only be measured in a stationary frame of reference. The cosmic microwave background (CMB) provides such a frame of reference, since we know that CMB radiation reaching us from all directions originated at the same time, to a very high degree of accuracy (assuming the “Big Bang” origin of the Universe). It would be possible to adjust the measurements of neutrino flight times obtained at OPERA to allow for the motion of the experiment (and entire Earth system) against the CMB frame of reference, which is known to a higher degree of accuracy than the experimental determination of neutrino velocity, without necessarily re-running the experiment. Additional simple statistical tests of the OPERA experiment data which could easily be carried out may also be able to determine whether the one-way light speed problem explains the superluminal neutrino velocity which was apparently observed.

2 Einstein’s Train Thought-Experiment

A consideration of Einstein’s train thought-experiment helps in understanding the practical difficulties in measuring the one-way speed of light.

2.1 The Relativity of Simultaneity

The thought-experiment requires us to imagine a train passing an observer on a platform at a velocity at which relativistic effects can be measured. The distance between the train and the platform is assumed to be negligible and the train is transparent such that an observer on the platform can measure phenomena inside the train. Fig 1 shows the three snapshots of not only what the observer on the platform, at B, would see (green), but also what an observer at the centre of the train, at Y, would observe (yellow) and, in Fig 1c, the perspective of an observer in a second train travelling in the opposite direction, at I (purple).

Fig 1a shows the centre of the train, Y, passing the observer on the platform at B at time t=0. It should be noted that the observer at B will observe the train to be red-shifted and longer in its direction of travel, towards C, and therefore that the front of the train, Z, had already passed C. Similarly, the train would appear blue-shifted and shorter towards A. The observer on the train, at Y, would see point C on the platform as blue-shifted and nearer than the front of the train, Z, and also point A on the platform as red-shifted and further away than the rear of the train, X. These aspects, though important, have been omitted from the diagram for clarity.

The thought-experiment requires a flash of light to be emitted at the centre of the train, Y, at the exact moment that the Y passes the observer on the platform at B. The green and yellow lines within the train represent the simplest form of the thought-experiment,
that is, the relativity of simultaneity. The observer in the train at Y sees the flash of light propagating at the same velocity in all directions (shown in yellow), so that, as shown in Fig 1c the light reaches both ends of the train simultaneously. The observer on the platform, at B, however, sees the light (shown in green) reaching the rear of the train before it reaches the front of the train.

In this paper, the thought-experiment is elaborated. At the moment when the centre of the train passes the observer on the platform at B a second flash of light is emitted from B, simultaneous to that emitted within the train at Y. The propagation of this flash of light in both directions along the platform is also shown in Figs 1b and 1c, in yellow and green representing the perspectives of the observers at Y and B, respectively.

The relativity of simultaneity is again apparent. The observer on the platform at B sees the light simultaneously reach the points A and C, which are equidistant from B and represent the ends of a stationary train at the platform. The observer on the train at Y, however, sees the light reaching C before it reaches A.

The observations of the flash of light emitted on the platform are exactly analogous to those of the flash of light emitted on the train. That is, the observers on the train and on the platform are simply moving relative to each other. The diagrams could be relabelled
to show ABC as a train moving left and XYZ as a platform, rather than ABC as the platform and XYZ as a train moving right.

Now, consider the green lines representing the view of the observer on the platform at B of flashes of light emitted simultaneously on the train and on the platform. It will be seen that B sees light moving at the same velocity in both directions in both frames of reference. If this were not so then it would be trivial (at least in a thought-experiment) to arrange faster than light communication. If light were travelling faster inside the train towards its rear at A than along the platform towards X, as might be presumed from the shortening (accompanied by blue-shifting) of the rear of the train as seen from B, then the observer at B could simply send a message to Y to be sent to X, then to the platform and on to A (perhaps by the use of appropriately angled mirrors).

This is confirmed by the fact that light is observed to arrive from distant sources at the same velocity whether it is blue-shifted, from a source, such as a star or galaxy, moving towards us, or red-shifted, from a source moving away from us.

Therefore, assuming special relativity, the one-way speed of light connects different frames of reference. Special relativity would be falsified if some light was ever observed to travel even infinitesimally faster than other light (in the same medium, in the same direction and under the same gravitational influences), however fast or slow the object that emitted it was moving relative to the observer.

2.2 Adjusting Clocks Between Different Frames of Reference

The relativity of simultaneity indicates the difficulties in synchronising measurements of time between different frames of reference. The observer on the platform at B may adopt a convention for clocks on the platform at A, B and C. Simplest is to set all these to the same time, $T_B$, so that the flash of light emitted on the platform reaches A and C at $T_B + d/c$, where $d$ is the equal distance AB and BC, and $c$ the speed of light (as seen by all observers).

But if the observer at the centre of the train, Y, synchronises their clock to that at B ($T_Y = T_B$) then the time when the flash of light as seen from Y reaches X and Z is ($T_Y + d/c$) only within the XYZ frame of reference. If Y wishes to know the time at which the light reaches the ends of the train as perceived by B, then special relativity concludes that the clock at the front of the train, Z, needs to be adjusted such that:

$$T_Z = T_C + (vd/c^2)/\sqrt{1-v^2/c^2}$$

where $d$ is the distance YZ and $v$ the velocity of the train XYZ relative to the platform ABC. Note that in this example the convention has been adopted such that $T_C = T_B = T_Y$, so the light would be perceived to reach the front of the train, Z, by an observer at point B at:

$$L_{ZB} = L_{YB} + d/c + (vd/c^2)/\sqrt{1-v^2/c^2}$$

where $L_{YB}$ and $L_{ZB}$ are the specific times at which the light was emitted at Y and reached Z, respectively, as perceived by an observer at B.
Similarly the clock at X needs to be adjusted by 
\[-\frac{(vd/c^2)}{\sqrt{1-v^2/c^2}}\]

for the observer in the train to determine when events occur in the ABC timeframe.

The need to adjust clocks between different frames of reference is already well understood. The Sagnac Effect occurs in rotating frames of reference such as the Earth system (ES). Clocks, for example in the Global Positioning System (GPS) need to be adjusted, [4], to take account of the fact that if you circumnavigate the globe from west to east you lose time (about 200ns at the Equator) compared to stationary clocks. Of course, no clock adjustment would be necessary were the Earth not rotating, since you won’t end up back at your starting point in a flat frame of reference, such as those in Fig 1.

2.3 Multiple Frames of Reference

Fig 1c shows a second train on the other side of the platform, the centre, I, of which passed points B and Y simultaneously at the moment light flashes were emitted from B and Y. A light flash was also emitted from point I at the same instant.

The purple lines indicate the positions of the three light flashes as perceived by an observer at I. Neither events simultaneous (the light reaching X and Z) in the frame of reference XYZ, nor those (the light reaching A and C) in the frame of reference ABC, appear simultaneous at I. Nor would the reverse be true, that is, the light emitted at I would not appear to reach H and J simultaneously either from Y (reading down from the yellow lines in Fig 1c) or B (reading down from the green lines in Fig 1c).

So, in all frames of reference moving relative to each other we are forced to adopt a convention, [2], for the measurement of time. It is only because we generally adopt a convention ignoring (or simplifying) the relationship between time, c and distance (except for the Sagnac Effect) that we measure a constant speed of light in all directions.

The question arises as to whether there is a stationary frame of reference against which all clocks can be adjusted.

3 The CMB as a Stationary Frame of Reference

The CMB radiation we observe today in all directions in space was emitted simultaneously in the Big Bang ~13.7bn years ago, that is, from the surface of a sphere of space centred on the Earth that was at that time ~13.7bn light years away from our present location. This is how we know we’re at the point in time ~13.7bn years after the origin of the Universe.

Every object in space continually sees CMB radiation emitted from the surface of different, generally overlapping, spheres, because only radiation that has had exactly the right amount of time to reach them at the constant speed of light, c, will do so at any given moment.

According to cosmological theory, the CMB radiation was emitted simultaneously everywhere in all directions, but what is reaching us now is precisely that which was ~13.7bn light years away from our present position when it was emitted. Other CMB radiation emitted in our direction has either reached us already or has yet to reach us, depending on the distance at which it was emitted from our present position.
Since we have shown that special relativity would be falsified if we perceived light to ever travel even infinitesimally faster than \( c \), then adjusting our clocks relative to the CMB frame of reference, that is, adjusting for the Earth’s motion relative to the CMB, will permit time measurements relative to a single known event that occurred at a single moment in time, that is, the Big Bang.

It should also be noted that were we to emit light that could travel faster than the CMB radiation in any direction then there would be a potential for causality violation, that is, the light (or any other object or particle) would eventually reach a region of space where the Big Bang was still in progress and potentially interfere with its own creation. Given the relationship between time and distance highlighted by the relativity of simultaneity, superluminal travel would be equivalent to travel back in time and \textit{inter alia} incompatible with the Big Bang theory.

Further, if the CMB was not all emitted at the same time, then the Big Bang theory is at the very least undermined.

Measurements of the red-shift/blue-shift dipole of the CMB suggest a motion relative to the CMB of the Earth (and the rest of the Solar System) of \( \sim 600\text{km/s} \) \((\sim 0.002c)\). Other components of the Earth’s motion, such as its rotation (<1km/s even at the Equator) and the Earth’s orbit about the Sun \((\sim 30\text{km/s})\), are fairly insignificant in comparison with the motion relative to the CMB, though they affect the orientation of the planet in relation to the CMB over relatively short timescales.

Given the Earth’s absolute velocity, \( v_e \), of \( \sim 0.002c \), the difference from unity of the denominator in (1) is insignificant. Therefore, to a reasonable approximation, the relationship between time measurements in an experiment in the Earth’s frame of reference, \( T_{\text{Exp}} \) (rather than \( T_{\text{Earth}} \) to emphasise that the same adjustment does not apply to the whole planet) to those in that of the CMB (henceforth “absolute time”, \( T_{\text{ABS}} \)) is given by:

\[
T_{\text{Exp}} = T_{\text{ABS}} \pm v_e d/c^2 \tag{3}
\]

that is, we can say that as an approximation it will be necessary to adjust Earthbound clocks by up to 5.6ns/km, in order to convert measurements of the one-way speed of light to \( c \):

\[
5.6\text{ns/km} \geq (T_{\text{Exp}} - T_{\text{ABS}}) \geq -5.6\text{ns/km} \tag{4}
\]

The sign and magnitude (up to this maximum value) of the actual relationship at any moment will depend on the orientation of the specific direction of the motion being measured (not of the whole Earth) in relation to the CMB. It is feasible to establish \((T_{\text{Exp}} - T_{\text{ABS}})\) for a given experimental configuration to accurately measure particle velocities relative to \( c \), although the adjustment \((T_{\text{Exp}} - T_{\text{ABS}})\) will (in a static experimental set-up) continually vary with time because of the Earth’s rotation and orbit about the Sun.
4 Implications for the OPERA Neutrino Speed Measurement

For most purposes, the two-way light speed, $c$, is sufficient. In other applications (such as the GPS) implicit one-way light speed measurements are used in ways that do not introduce errors.

The set-up for the OPERA neutrino speed measurement, [1], however, included measurements that require an adjustment of up to $\pm 5.6\text{ns/km}$, (4). These may have at least contributed towards the observed neutrino velocity of $\sim 1.000025c$, since, based on the description in [1], these measurements were made over a number of kilometres so, at up to $5.6\text{ns/km}$ may sum to the observed discrepancy of $60\text{ns}$ in neutrino flight times compared to $c$, given that the sign of the measurement errors are likely to almost all the same as the experiment is a linear one.

4.1 GPS Measurements

The GPS makes no allowance for the relativity of simultaneity. In other words, it is based on one-way light speed measurements. This introduces no errors, however, since distances as measured in the ES frame of reference are also affected, as discussed in the context of Einstein’s train thought-experiment, such that we always perceive light to travel at $c$.

The GPS measures relative positions based on Coordinated Universal Time (UTC), which takes account of adjustments for many potential sources of inaccuracy, including the Sagnac Effect. Positions and times are all measured within the same ES frame of reference, so are mutually consistent. That is, distances are unaffected by the effect of the $\sim 0.002c$ velocity of the Earth relative to the CMB (and the planet’s continually changing orientation with respect to its direction of motion). An observer at rest relative to the CMB would see light travelling at different velocities along and against the direction of motion of the Earth, and would also see distances shortened and blue-shifted on approaching parts of the planet, and lengthened and red-shifted on receding parts, but such considerations do not affect measurements taken entirely within the ES frame of reference. This is similar to the situation in the thought-experiment, Fig 1, where valid distance measurements within either train or along the platform could be made from Y and B respectively, using the observed speed of light propagation as a “ruler”.

The OPERA neutrino experiment aims to test the null hypothesis that neutrinos travel at the speed of light. Even if the one-way propagation of light “should” in fact be measured in another frame of reference, such as that provided by the CMB, the same would apply to the neutrinos. If the neutrino flight path had been precisely between the two accurate GPS receivers used, then a true measurement of their velocity (relative to $c$) would have been obtained.

Unfortunately this was not the case since the neutrino detector is underground at some distance from the GPS receiver.

4.2 Two-Way Light Speed Measurements at CERN and OPERA

Two procedure(s) were used to measure timing signal transmission times between the GPS receivers at CERN and OPERA and the points where neutrinos were originated and detected. Based on the description in [1], the most significant errors are at OPERA, in
particular related to transmission of a signalling pulse from the GPS clock to the neutrino detector:

“Every millisecond a pulse synchronously derived from the 1PPS of the ESSat (PPmS) is transmitted to the underground laboratory via an 8.3 km long optical fibre. The delay of this transmission with respect to the ESAT 1PPS output down to the OPERA master clock output was measured with a two-way fibre procedure and amounts to (40996 ± 1) ns. Measurements with a transportable Cs clock were also performed yielding the same result.” ([1], p.13, my stress).

Simply moving a clock for the purpose of measuring a time delay is entirely different to using the GPS to establish simultaneity within the ES frame of reference. The GPS works because one-way light speed (and distance) measurements cancel out. Referring again to Fig 1, moving the clock between two points establishes (in this case) a nominal time delay in the ES frame of reference, but we are comparing this against a phenomenon (one-way neutrino flight time) for which we do not know the correct frame of reference. The suggestion in this paper is that one-way light (and neutrino) speed measurements will only be consistent with two-way speed measurements in a frame of reference stationary with respect to the CMB. The time delay measured by moving the clock is therefore erroneous, or at least cannot be directly compared with the neutrino flight time.

To validate this conclusion it is also helpful to consider the two-way fibre procedure referred to. This is described as follows in [1], with my attempts to clarify included in square brackets (confusion arises since both “optical fibre B” and “direct path A” are optical fibre links, though the direct path A for which the procedure is attempting to establish a delay is likely to include other experimental components):

“The two-way measurement is a technique used several times in this analysis for the determination of delays. Measuring the delay t_A in propagating a signal to a far device consists in sending the same signal via an[other fibre link,] optical fibre B[,] to the far device location in parallel to its direct path A. At this site the time difference t_A–t_B between the signals following the two paths is measured. A second measurement is performed by taking the signal arriving at the far location via its direct path A and sending it back to the origin with the optical fibre B. At the origin the time difference between the production and receiving time of the signal corresponds to t_A+t_B. In this procedure the optoelectronic chain used for the fibre transmission of the two measurements is kept identical by simply swapping the receiver and the transmitter between the two locations. The two combined measurements allow determining t_A and t_B.” ([1], p.12).

That is, they simply add (t_A–t_B) and (t_A+t_B) and divide by 2 to yield t_A. This understanding is confirmed by [5], which is referenced in [1].

Unfortunately, though, the two-way fibre procedure assumes t_B in the measurement (t_A–t_B), henceforth t_B1, is exactly the same as t_B in the measurement (t_A+t_B), henceforth t_B2. This is only the case for phenomena measured in the ES frame of reference. In any frame of reference in motion relative to the ES, such as that of the CMB, t_B1 ≠ t_B2.

Two-way light speed measurement along the calibration fibres have been compared
against a one-way neutrino velocity measurement. This is a source of error in the neutrino velocity measurement which may explain some or all of the apparent superluminal neutrino observation.

5 Correcting the OPERA Neutrino Speed Measurements

There are two ways of testing the hypothesis presented in this paper, which is that neutrinos travel at c and that this can be verified by adjusting the frame of reference to that of the CMB. (The hypothesis could also be tested by experiments comparing simultaneous neutrino and light flight times, but facilities to do this do not yet exist). The tests depend on specific characteristics of the measurement error.

5.1 Common Component of Excess Neutrino Velocity

A statistical error would be expected depending on a common component of the experiment’s motion with respect to the CMB frame of reference. This arises from the fact that “about 16,000” neutrino timing measurements were taken over several years. If the hypothesis presented in this paper is correct, then the result of \((v-c)/c\), where \(v\) is the observed neutrino velocity, of \(~0.000025\) is a result of the average motion of the experiment relative to the CMB, i.e. the North-South component of the Earth’s motion relative to the CMB. Of course the error only occurred over part of the neutrino flight path. If this distance, \(d\), can be determined it should be possible to make an adjustment by simply calculating the velocity of the component of the Earth’s motion along its axis in relation to the CMB redshift/blueshift dipole, which is already well constrained by astronomical observation, multiplying this to \(d\), and then adjusting \(v\) in proportion to \(d/d_{\text{tot}}\) where \(d_{\text{tot}}\) is the total length of the neutrino flight path in the experiment (~730km). The hypotheses suggested in this paper \((v=c)\ and that one-way light speed measurements yield c in the CMB frame of reference) would be confirmed if \(v\) is no longer found to be significantly different from \(c\) once the adjustment has been made.

5.2 Variable Component of Excess Neutrino Velocity

The design of the OPERA experiment did not directly measure the velocity of individual neutrinos. Rather the result is based on a statistical analysis. However, tests of possible time of day and time of year effects were carried out:

“Data were also grouped in arbitrary subsamples to look for possible systematic dependences. For example, by computing \(\delta t\) separately for events taken during day and night hours, the absolute difference between the two bins is \((17.1 \pm 15.5)\) ns providing no indication for a systematic effect. A similar result was obtained for a possible summer vs spring + fall dependence, which yielded \((11.3 \pm 15.5)\) ns.” ([1], p.15). [sic, winter is not mentioned – perhaps the experiment was not run in winter].

If the hypothesis in this paper is correct, then the experiment would be expected to observe \((v-c)/c\) to be greater when the experiment is orientated such that the neutrino flight path is towards the area where the CMB appears most blue-shifted (see Fig 1). The orientation of the experiment in space will vary depending on both the Earth’s
rotation and its orbit about the Sun. It would a simple matter to test other possible “systematic dependences” such as a comparison of subsamples of events taken during the day in spring vs during the night in autumn and vice versa. A significant difference between any two such bins would tend to support this paper’s hypothesis.

5.3 Other Possible Errors

This paper argues that the source of the errors in the OPERA neutrino velocity experiment is the fibre delay calibration procedure used, as described in [1] and [5]. If this is the case, and a similar procedure has been used in other experiments, then obviously further errors in velocity measurement and other timings may have occurred.

Finally, it should not be assumed that the invalid fibre delay calibration procedure used in the OPERA neutrino velocity measurement experiment is the only error that may have occurred. Depending on distance and other measurements, it may be the case that the invalid fibre delay calibration procedure explains only part of the ~60ns early neutrino arrival at OPERA.
References


## Modification History

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